Electric Power Distribution Systems II

EME410

Spring 21-22

Lecture 10

A.C. Distribution

INSTRUCTOR

DR / AYMAN SOLIMAN

➢ Contents

- Introduction
- A.C. Distribution Calculations
- Methods of Solving A.C. Distribution Problems
- (i) Power factors referred to receiving end voltage.
- (ii) Power factors referred to respective load voltag
- Examples

Introduction

- In the beginning of the electrical age, electricity was generated as a direct current and voltages were low.
- The principal disadvantage of d.c. system was that voltage level could not readily be changed, except by the use of rotating machinery, which in most cases was too expensive. With the development of the transformer, a.c. has taken over the load formerly supplied by d.c.
- Now-a-days, electrical energy is generated, transmitted and distributed in the form of a.c. as an economical proposition.

Introduction (cont.)

- The electrical energy produced at the power station is transmitted at very high voltages by 3-phase, 3- wire system to step-down sub-stations for distribution. The distribution system consists of two parts viz. primary distribution and secondary distribution.
- The primary distribution circuit is 3-phase, 3-wire and operates at voltages (3·3 or 6·6 or 11kV) somewhat higher than general utilization levels. It delivers power to the secondary distribution circuit through distribution transformers situated near consumers' localities.

➢ A.C. Distribution Calculations

- ➢ A.C. distribution calculations differ from those of d.c. distribution in the following respects :

(i) In case of d.c. system, the voltage drop is due to resistance alone. However, in a.c. system, the voltage drops are due to the combined effects of resistance, inductance and capacitance.

(ii) In a d.c. system, additions and subtractions of currents or voltages are done arithmetically but in case of a.c. system, these operations are done vectorially.

(iii) In an a.c. system, power factor (p.f.) has to be taken into account. Loads tapped off form the distributor are generally at different power factors.

A.C. Distribution Calculations (cont.)

- There are two ways of referring power factor viz

(a) It may be referred to supply or receiving end voltage which is regarded as the reference vector.

(b) It may be referred to the voltage at the load point itself.

➢ Methods of Solving A.C. Distribution Problems

- In a.c. distribution calculations, power factors of various load currents have to be considered since currents in different sections of the distributor will be the vector sum of load currents and not the arithmetic sum.

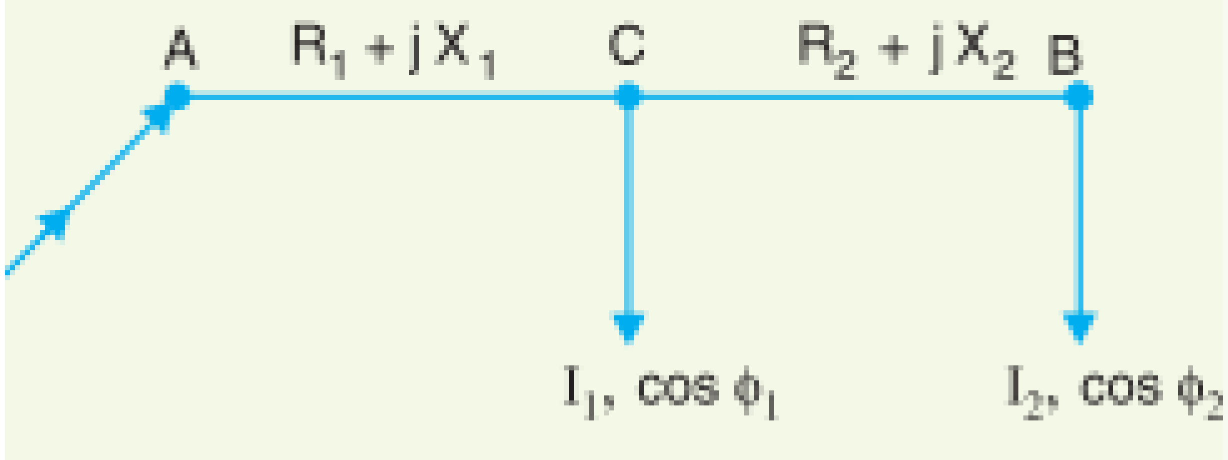
- The power factors of load currents may be given

(i) w.r.t. receiving or sending end voltage or

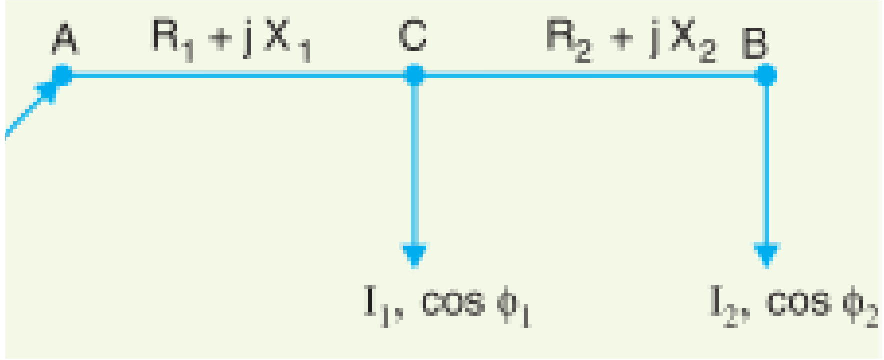
(ii) w.r.t. to load voltage itself.

➢ (i) Power factors referred to receiving end voltage.

- ➢ Consider an a.c. distributor A B with concentrated loads of I1 and I2 tapped off at points C and B as shown in Fig. 14.1. Taking the receiving end voltage V B as the reference vector, let lagging power factors at C and B be cos ϕ1 and cos ϕ2 w.r.t. V B. Let R 1, X 1 and R2, X 2 be the resistance and reactance of sections AC and CB of the distributor.



➢ (i) Power factors referred to receiving end voltage.



Impedance of section AC, $\overrightarrow{Z_{AC}} = R_1 + jX_1$

Impedance of section CB, $\overrightarrow{Z_{CB}} = R_2 + jX_2$

Load current at point C, $\overrightarrow{I_1} = I_1(\cos\phi_1 - j\sin\phi_1)$

Load current at point B, $\overrightarrow{I_2} = I_2(\cos\phi_2 - j\sin\phi_2)$

Current in section CB, $\overrightarrow{I_{CB}} = \overrightarrow{I_2} = I_2(\cos\phi_2 - j\sin\phi_2)$

Current in section AC, $\overrightarrow{I_{AC}} = \overrightarrow{I_1} + \overrightarrow{I_2}$

$= I_1(\cos\phi_1 - j\sin\phi_1) + I_2(\cos\phi_2 - j\sin\phi_2)$

Voltage drop in section CB, $\overrightarrow{V_{CB}} = \overrightarrow{I_{CB}}\,\overrightarrow{Z_{CB}} = I_2(\cos\phi_2 - j\sin\phi_2)(R_2 + jX_2)$

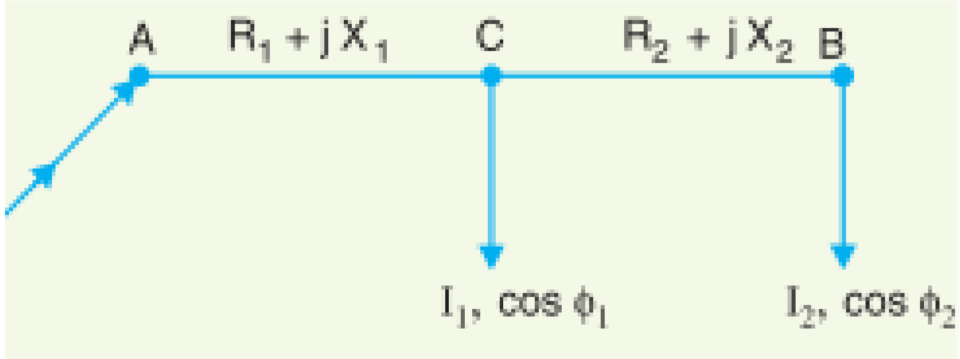
Voltage drop in section AC, $\overrightarrow{V_{AC}} = \overline{I_{AC}}\,\overline{Z_{AC}} = \left(\overline{I_1} + \overline{I_2}\right) Z_{AC}$

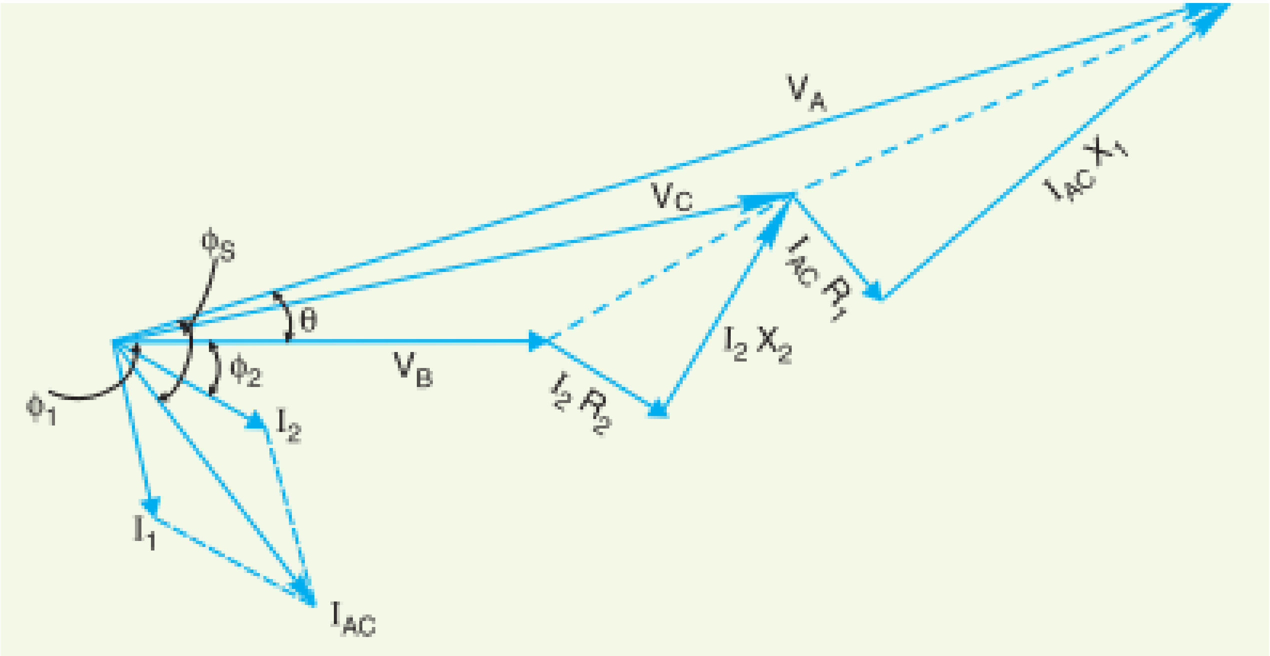
$= [I_1(\cos\phi_1 - j\sin\phi_1) + I_2(\cos\phi_2 - j\sin\phi_2)]\,[R_1 + jX_1]$

Sending end voltage, $\overrightarrow{V_A} = \overrightarrow{V_B} + \overrightarrow{V_{CB}} + \overrightarrow{V_{AC}}$

Sending end current, $\overrightarrow{I_A} = \overrightarrow{I_1} + \overrightarrow{I_2}$

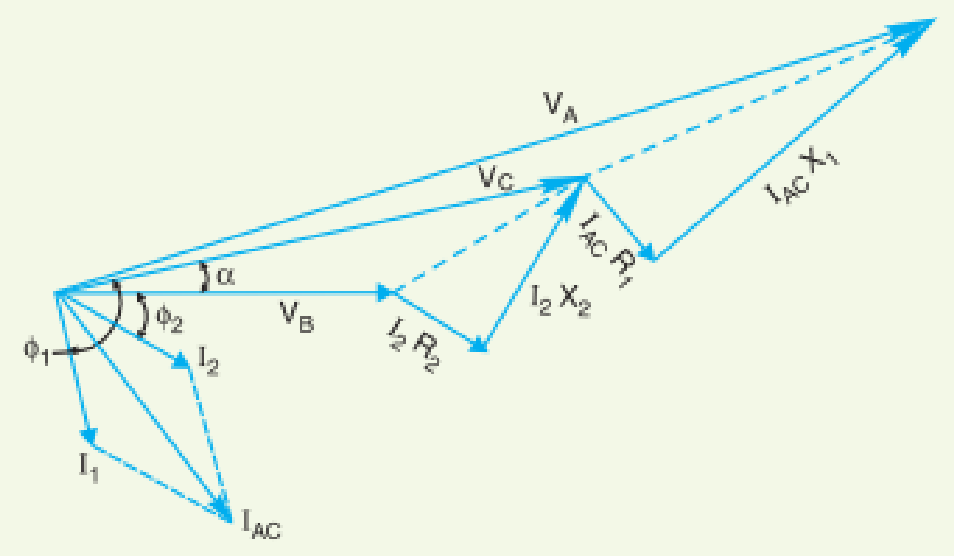
➢ (i) Power factors referred to receiving end voltage.



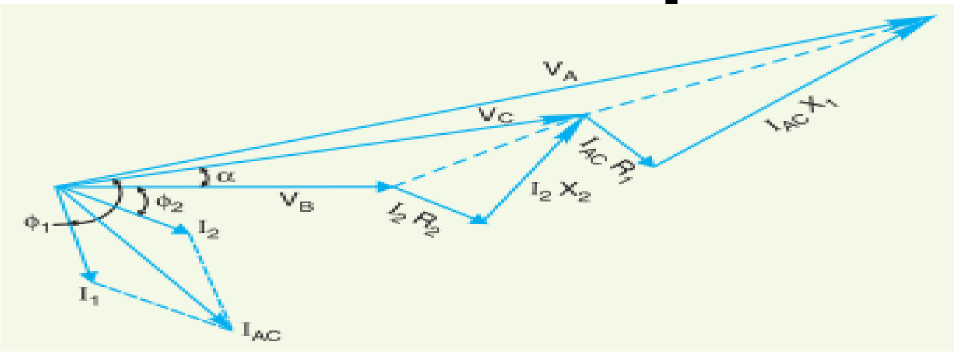


➢ (ii) Power factors referred to respective load voltages.

Suppose the power factors of loads in the previous Fig. are referred to their respective load voltages. Then ϕ1 is the phase angle between VC and I1 and ϕ2 is the phase angle between VB and I2. The vector diagram under these conditions is shown in Fig



➢ (ii) Power factors referred to respective load voltages.



Voltage drop in section CB $= \vec{I_2}\,\vec{Z_{CB}} = I_2(\cos\phi_2 - j\sin\phi_2)(R_2 + j X_2)$

Voltage at point C $= \vec{V_B}$ + Drop in section $CB = V_C \angle \alpha$ (say)

Now $\vec{I_1} = I_1 \angle -\phi_1$ *w.r.t.* voltage V_C

$\therefore$ $\vec{I_1} = I_1 \angle -(\phi_1 - \alpha)$ *w.r.t.* voltage V_B

i.e. $\vec{I_1} = I_1 [\cos(\phi_1 - \alpha) - j\sin(\phi_1 - \alpha)]$

Now $\vec{I_{AC}} = \vec{I_1} + \vec{I_2}$

$= I_1[\cos(\phi_1 - \alpha) - j\sin(\phi_1 - \alpha)] + I_2(\cos\phi_2 - j\sin\phi_2)$

Voltage drop in section AC $= \vec{I_{AC}}\,\vec{Z_{AC}}$

$\therefore$ Voltage at point A $= V_B$ + Drop in CB + Drop in AC

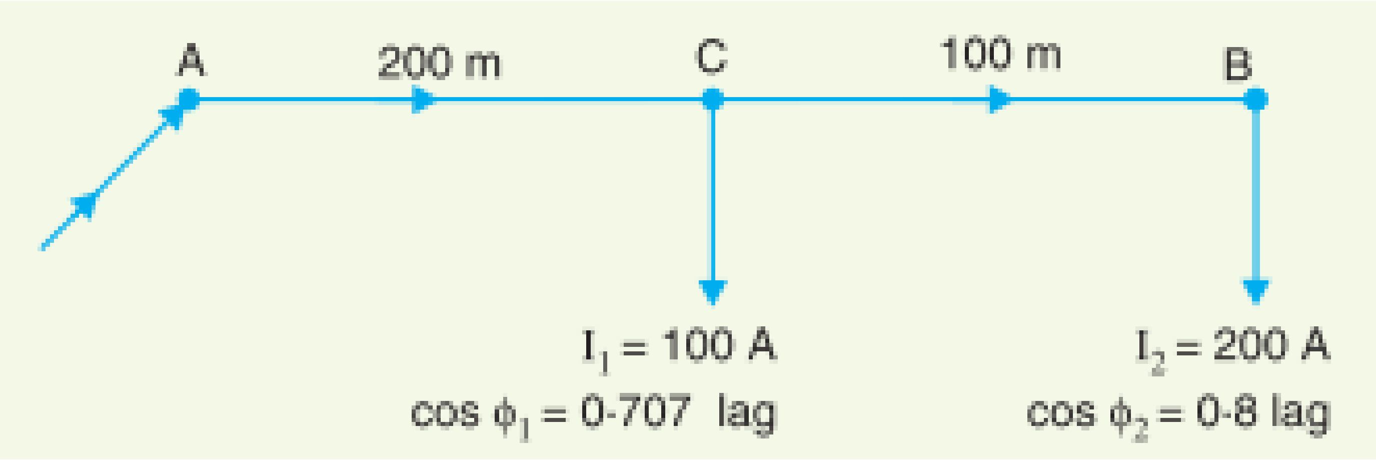
➢ Example 1

A single phase a.c. distributor AB 300 metres long is fed from end A and is loaded as under :

(i) 100 A at 0·707 p.f. lagging 200 m from point A

(ii) 200 A at 0·8 p.f. lagging 300 m from point A

The load resistance and reactance of the distributor is 0·2 Ω and 0·1 Ω per kilometre. Calculate the total voltage drop in the distributor. The load power factors refer to the voltage at the far end.



➢ Example 1 (cont.)

Impedance of section AC, $\overrightarrow{Z_{AC}} = (0{\cdot}2 + j\,0{\cdot}1) \times 200/1000 = (0{\cdot}04 + j\,0{\cdot}02)\,\Omega$

Impedance of section CB, $\overrightarrow{Z_{CB}} = (0{\cdot}2 + j\,0{\cdot}1) \times 100/1000 = (0{\cdot}02 + j\,0{\cdot}01)\,\Omega$

Taking voltage at the far end B as the reference vector, we have,

Load current at point B, $\overrightarrow{I_2} = I_2(\cos\phi_2 - j\sin\phi_2) = 200\,(0{\cdot}8 - j\,0{\cdot}6)$
$= (160 - j\,120)$ A

Load current at point C, $\overrightarrow{I_1} = I_1(\cos\phi_1 - j\sin\phi_1) = 100\,(0{\cdot}707 - j\,0{\cdot}707)$
$= (70{\cdot}7 - j\,70{\cdot}7)$ A

Current in section CB, $\overrightarrow{I_{CB}} = \overrightarrow{I_2} = (160 - j\,120)$ A

Current in section AC, $\overrightarrow{I_{AC}} = \overrightarrow{I_1} + \overrightarrow{I_2} = (70{\cdot}7 - j\,70{\cdot}7) + (160 - j\,120)$
$= (230{\cdot}7 - j\,190{\cdot}7)$ A

Voltage drop in section CB, $\overrightarrow{V_{CB}} = \overrightarrow{I_{CB}}\,\overrightarrow{Z_{CB}} = (160 - j\,120)\,(0{\cdot}02 + j\,0{\cdot}01)$
$= (4{\cdot}4 - j\,0{\cdot}8)$ volts

Voltage drop in section AC, $\overrightarrow{V_{AC}} = \overrightarrow{I_{AC}}\,\overrightarrow{Z_{AC}} = (230{\cdot}7 - j\,190{\cdot}7)\,(0{\cdot}04 + j\,0{\cdot}02)$
$= (13{\cdot}04 - j\,3{\cdot}01)$ volts

Voltage drop in the distributor $= \overrightarrow{V_{AC}} + \overrightarrow{V_{CB}} = (13{\cdot}04 - j\,3{\cdot}01) + (4{\cdot}4 - j\,0{\cdot}8)$
$= (17{\cdot}44 - j\,3{\cdot}81)$ volts

Magnitude of drop $= \sqrt{(17{\cdot}44)^2 + (3{\cdot}81)^2} =$ **17·85 V**

Example 2

A single phase distributor 2 kilometers long supplies a load of 120 A at 0·8 p.f. lagging at its far end and a load of 80 A at 0·9 p.f. lagging at its mid-point. Both power factors are referred to the voltage at the far end. The resistance and reactance per km (go and return) are 0·05 Ω and 0·1 Ω respectively. If the voltage at the far end is maintained at 230 V, calculate :

(i) voltage at the sending end

(ii) phase angle between voltages at the two ends.

➢ Example 2 (cont.)

Solution. Fig. 14.5 shows the distributor *AB* with *C* as the mid-point

Impedance of distributor/km $= (0{\cdot}05 + j\,0{\cdot}1)\,\Omega$

Impedance of section *AC*, $\vec{Z}_{AC} = (0{\cdot}05 + j\,0{\cdot}1) \times 1000/1000 = (0{\cdot}05 + j\,0{\cdot}1)\,\Omega$

Impedance of section *CB*, $\vec{Z}_{CB} = (0{\cdot}05 + j\,0{\cdot}1) \times 1000/1000 = (0{\cdot}05 + j\,0{\cdot}1)\,\Omega$

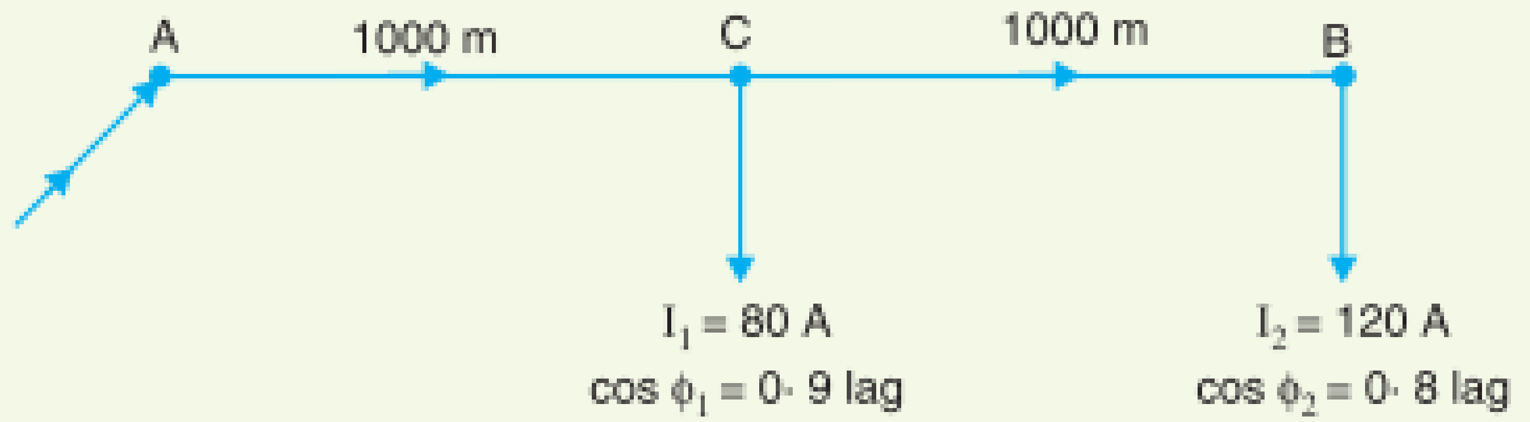


Fig. 14.5

➢ Example 2 (cont.)

Let the voltage V_B at point B be taken as the reference vector.

Then, $\vec{V_B} = 230 + j\,0$

(*i*) Load current at point B, $\vec{I_2} = 120\,(0{\cdot}8 - j\,0{\cdot}6) = 96 - j\,72$

Load current at point C, $\vec{I_1} = 80\,(0{\cdot}9 - j\,0{\cdot}436) = 72 - j\,34{\cdot}88$

Current in section CB, $\vec{I_{CB}} = \vec{I_2} = 96 - j\,72$

Current in section AC, $\vec{I_{AC}} = \vec{I_1} + \vec{I_2} = (72 - j\,34{\cdot}88) + (96 - j\,72)$

$= 168 - j\,106{\cdot}88$

Drop in section CB, $\vec{V_{CB}} = \vec{I_{CB}}\,\vec{Z_{CB}} = (96 - j\,72)\,(0{\cdot}05 + j\,0{\cdot}1)$

$= 12 + j\,6$

Drop in section AC, $\vec{V_{AC}} = \vec{I_{AC}}\,\vec{Z_{AC}} = (168 - j\,106{\cdot}88)\,(0{\cdot}05 + j\,0{\cdot}1)$

$= 19{\cdot}08 + j\,11{\cdot}45$

∴ Sending end voltage, $\vec{V_A} = \vec{V_B} + \vec{V_{CB}} + \vec{V_{AC}}$

$= (230 + j\,0) + (12 + j\,6) + (19.08 + j\,11.45)$

$= 261.08 + j\,17.45$

Its magnitude is $= \sqrt{(261{\cdot}08)^2 + (17{\cdot}45)^2} =$ **261·67 V**

(*ii*) The phase difference θ between V_A and V_B is given by :

$$\tan\theta = \frac{17{\cdot}45}{261{\cdot}08} = 0{\cdot}0668$$

∴ $\theta = \tan^{-1} 0{\cdot}0668 =$ **3·82°**

➢ Example 3

A 3-phase, 400V distributor AB is loaded as shown in Fig.14.8. The 3-phase load at point C takes 5A per phase at a p.f. of 0·8 lagging. At point B, a 3-phase, 400 V induction motor is connected which has an output of 10 H.P. with an efficiency of 90% and p.f. 0·85 lagging.

If voltage at point B is to be maintained at 400 V, what should be the voltage at point A ? The resistance and reactance of the line are 1Ω and 0·5Ω per phase per kilometer respectively.

➢ Example 3 (cont.)

Solution. It is convenient to consider one phase only. Fig.14.8 shows the single line diagram of the distributor. Impedance of the distributor per phase per kilometre = $(1 + j\,0{\cdot}5)\,\Omega$.

Impedance of section AC, $\overrightarrow{Z_{AC}} = (1 + j\,0{\cdot}5) \times 600/1000 = (0{\cdot}6 + j\,0{\cdot}3)\,\Omega$

Impedance of section CB, $\overrightarrow{Z_{CB}} = (1 + j\,0{\cdot}5) \times 400/1000 = (0{\cdot}4 + j\,0{\cdot}2)\,\Omega$

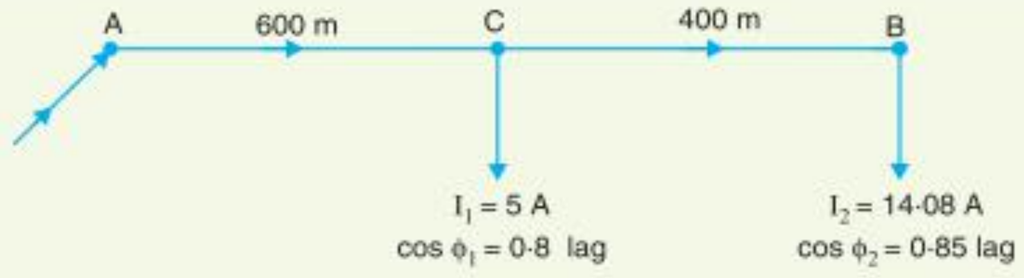


Fig. 14.8

Phase voltage at point B, $V_B = 400/\sqrt{3} = 231$ V

Let the voltage V_B at point B be taken as the reference vector.

Then, $\overrightarrow{V_B} = 231 + j\,0$

Line current at B

$$= \frac{\text{H.P.} \times 746}{\sqrt{3} \times \text{line voltage} \times \text{p.f.} \times \text{efficiency}}$$

$$= \frac{10 \times 746}{\sqrt{3} \times 400 \times 0{\cdot}85 \times 0{\cdot}9} = 14{\cdot}08 \text{ A}$$

∴ *Current/phase at B, $I_2 = 14{\cdot}08$ A

➢ Example 3 (cont.)

∴ *Current/phase at B, $I_2 = 14{\cdot}08$ A

Load current at B, $\vec{I_2} = 14{\cdot}08\,(0{\cdot}85 - j\,0{\cdot}527) = 12 - j\,7{\cdot}4$

Load current at C, $\vec{I_1} = 5\,(0{\cdot}8 - j\,0{\cdot}6) = 4 - j\,3$

Current in section AC, $\overline{I}_{AC} = \vec{I_1} + \vec{I_2} = (4 - j\,3) + (12 - j\,7{\cdot}4)$

$= 16 - j\,10{\cdot}4$

Current in section CB, $\overline{I}_{CB} = \vec{I_2} = 12 - j\,7{\cdot}4$

Voltage drop in CB, $\vec{V}_{CB} = \vec{I}_{CB}\vec{Z}_{CB} = (12 - j\,7{\cdot}4)\,(0{\cdot}4 + j\,0{\cdot}2)$

$= 6{\cdot}28 - j\,0{\cdot}56$

Voltage drop in AC, $\vec{V}_{AC} = \vec{I}_{AC}\vec{Z}_{AC} = (16 - j\,10{\cdot}4)\,(0{\cdot}6 + j\,0{\cdot}3)$

$= 12{\cdot}72 - j\,1{\cdot}44$

Voltage at A per phase, $\vec{V}_A = \vec{V}_B + \vec{V}_{CB} + \vec{V}_{AC}$

$= (231 + j\,0) + (6{\cdot}28 - j\,0{\cdot}56) + (12{\cdot}72 - j\,1{\cdot}44)$

$= 250 - j\,2$

Magnitude of V_A/phase $= \sqrt{(250)^2 + (2)^2} = 250$ V

∴ Line voltage at A $= \sqrt{3} \times 250 =$ **433 V**


Thank you